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(e)KENMOTSU MANIFOLD ADMITTING SCHOUTEN-VAN KAMPEN CONNECTION

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(ϵ)-KENMOTSU MANIFOLD ADMITTING SCHOUTEN-VAN KAMPEN CONNECTION

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Abstract. The purpose of this paper is to study various geometric properties of (ϵ)-Kenmotsu manifold admitting Schouten-van Kampen connection. A unique relation between curvature tensors of Schouten-van Kampen connection and Levi-Civita connection have been obtained. We study quasi-conformally flat as well as conformally flat (ϵ)-Kenmotsu manifold with respect to Schouten-van Kampen connection. Moreover, it is shown that a ϕ-conformally flat (ϵ)-Kenmotsu manifold with respect to Schouten-van Kampen connection is an η-Einstein manifold. Also, we study (ϵ)-Kenmotsu manifold with respect to Schouten-van Kampen connection satisfying $C^*(\xi, U).R^* = 0$ and $C^*(\xi, U).S^* = 0$, where C^*, R^*, S^* are conformal curvature tensor, Riemannian curvature tensor and Ricci tensor with respect to Schouten-van Kampen connection respectively.

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1. Introduction. The study of manifolds with indefinite metrics is of interest from the standpoint of physics and relativity. In 1969, Takahashi (1969) introduced almost contact metric manifold which is endowed with indefinite metrics and he proved that if a Sasakian manifold $M^{(2n+1)}$, $n \geq 1$ is complete, simply connected and is of constant ϕ-sectional curvature $\kappa \neq$-3, then it is D-homothetic to the model space $\bar{S}^{(2n+1)}_{2s}$ of Sasakian manifolds, where $2s$ = 'Sig g' if $\kappa > -3$ and $2s = 2n$ - 'Sig$g$' if $\kappa < -3$ (Bejancu and Faran, 2006). Manifolds with indefinite metrics have been studied by several authors. In 1993, Bejancu and Duggal (Bejancu, Duggal, 1993) introduced the concept of (ϵ)-Sasakian manifolds and Xufeng and Xiaoli (1998) established that these

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manifolds are real hypersurfaces of indefinite Kahlerian manifolds. In 1972, Kenmotsu (1972) studied a class of almost contact Riemannian manifold and introduced Kenmotsu manifold. De and Sarkar (2009) introduced (ϵ)-Kenmotsu manifolds and studied conformally flat, weyl semi-symmetric, ϕ-recurrent (ϵ)-Kenmotsu manifolds. (ϵ)-Kenmotsu manifolds have been studied by Singh et al. (2014), Prakasha, Malingappa and Mirji (2016), Venkatesha and Vishnudharan (2017), Haseeb and De (2019), Pandey et al. (2019) and many other. The notion of almost paracontact structure (ϕ, ξ, η) on a differentiable manifold was introduced by I. Sato (1976, 1977). In 1985, Kaneyuki and Kozai (1985) introduced a class of affine symmetric spaces, called parahermitian symmetric spaces, a paracomplex analogue of hermitian symmetric spaces. They gave the infinitesimal classification of parahermitian symmetric spaces with semi-simple automorphism groups, upto paraholomorphic equivalences. Paracontact metric manifolds have been studied by several authors (Dacko, 2004, Montano, Erken and Murathan,, 2012 and Welyezko, 2012).

The notion of Schouten-van Kampen connection have been introduced in the third decade of last century for the study of non-holomorphic manifold (Schouten and Van Kampen, 1930 and Vrinceanu, 1931). Schouten-van Kampen connection is one of the most natural connection on differentiable manifold endowed with an affine connection (Bejancu and Faran, 2006). In 2006, Bejancu (2006) studied some properties of Schouten-van Kampen connection on foliated manifold. After that Olszak (2013) studied Schouten-van Kampen connection on almost contact metric structure and showed some interesting results. Schouten-van Kampen connection is defined by (Schouten and Van Kampen, 1930)

$$\nabla^*{}_X Y = \nabla_X Y - \eta(Y)\nabla_X \xi - (\nabla_X \eta)(Y)\xi, \tag{1.1}$$

for all $X,\ Y,\ Z \in \chi(M)$.

Hermann Weyl tried to unify electromagnetism and gravitation, this attempt was carried out in 1919 (Weyl, 1919), however this attempt failed, but Weyl introduced a new principle of invariance in his theory, a scale change of the metric tensor given by

$$\bar{g} = e^{2\omega} g,$$

where $g, e, \bar{g}$ are metric tensors on the manifold M^n and $\omega = \omega(x)$ is a real function.

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This principle later evolved through Quantum field theory being known as gauge invariance (Jackson and Okun, 2001). A scale change of the metric tensor is classified as conformal transformation in Riemannian geometry leading to the Weyl conformal tensor (Wytler Cordeiro dos Santos, 2020). The expression for the conformal curvature tensor on a Riemannian manifold is given as

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{(n-2)}[g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y] + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y], \qquad (1.2)$$

for all $X,\ Y,\ Z \in \chi(M^n)$, where S and Q denotes Ricci tensor and Ricci operator respectively and r is the scalar curvature tensor.

The conformal curvature tensor on (ϵ)-Kenmotsu manifold with respect to Schouten-van Kampen connection is as follows

$$C^*(X,Y)Z = R^*(X,Y)Z - \frac{1}{(n-2)}[g(Y,Z)Q^*X - g(X,Z)Q^*Y + S^*(Y,Z)X - S^*(X,Z)Y] + \frac{r^*}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y], \qquad (1.3)$$

for all $X,\ Y,\ Z \in \chi(M^n)$, where C^*, R^*, S^*, Q^* and r^* denotes the conformal curvature tensor, Riemannian curvature tensor, Ricci tensor, Ricci operator and scalar curvature tensor with respect to Schouten-van Kampen connection respectively.

DEFINITION 1.1 *An n-dimensional (ϵ)-Kenmotsu manifold M^n is said to be $\eta-$Einstein manifold if the Ricci tensors S is of the form $S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y)$, for all $X,\ Y \in \chi(M)$, where a and b are scalars.*

DEFINITION 1.2 *An n-dimensional (ϵ)-Kenmotsu manifold M^n is said to be an Einstein manifold if the Ricci tensor is of the form $S(X,Y) = ag(X,Y)$, where a is a scalar function.*

2. (ϵ)-Kenmotsu Manifolds. An n-dimensional smooth manifold (M^n, g) is called an (ε)-almost contact metric manifold if

$$\phi^2 X = -X + \eta(X)\xi \qquad (2.1)$$

$$\eta(\xi) = 1, \qquad (2.2)$$

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$$g(\xi, \xi) = \epsilon, \tag{2.3}$$

$$\eta(X) = \epsilon g(X, \xi), \tag{2.4}$$

$$g(\phi X, \phi Y) = g(X, Y) - \epsilon\eta(X)\eta(Y), \tag{2.5}$$

where ϵ is 1 or -1 according as ξ is space-like or time-like and rank (ϕ)=n-1.

It is important to mention that in the above definition ξ is never a light-like vector field. If

$$d\eta(X, Y) = g(X, \phi Y) \tag{2.6}$$

for every X, Y $\in TM^n$, then we say that M^n is an (ϵ)-contact metric manifold. Also,

$$\phi\xi = 0 \quad \text{and} \quad \eta o \phi = 0. \tag{2.7}$$

If an (ϵ) contact metric manifold satisfies

$$(\nabla_X \phi)(Y) = -g(X, \phi Y)\xi - \epsilon\eta(Y)\phi X, \tag{2.8}$$

where ∇ denotes the Riemannian connection, then M^n is called an (ϵ)-Kenmotsu manifold (De and Sarkar, 2019). An (ϵ)-almost contact metric manifold is an (ϵ)-Kenmotsu manifold if and only if

$$\nabla_X \xi = \epsilon(X - \eta(X)\xi). \tag{2.9}$$

In an (ϵ)-Kenmotsu manifold, following relations hold

$$(\nabla_X \eta)(Y) = g(X, Y) - \epsilon\eta(X)\eta(Y), \tag{2.10}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.11}$$

$$R(\xi, X)Y = \eta(Y)X - \epsilon g(X, Y)\xi, \tag{2.12}$$

$$R(\xi, Y)Z = -\eta(Z)X + \epsilon g(X, Z)\xi, \tag{2.13}$$

$$S(X, \xi) = -(n-1)\eta(X), \tag{2.14}$$

$$S(\phi X, \phi Y) = S(X, Y) + \epsilon(n-1)\eta(X)\eta(Y). \tag{2.15}$$

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The Schouten-van Kampen connection (Zamkovoy, 2008) on an (ϵ)-Kenmotsu manifold is defined by

$$(\nabla^*{}_X Y) = \nabla_X Y + g(X,Y)\xi - \epsilon\eta(Y)X. \tag{2.16}$$

3. Curvature Properties of (ϵ)-Kenmotsu Manifold Admitting Schouten-van Kampen Connection. The curvature tensor R^* of Riemannian manifold M^n with respect to Schouten-van Kampen connection ∇^* is given by

$$R^*(X,Y)Z = \nabla^*{}_X\nabla^*{}_Y Z - \nabla^*{}_Y\nabla^*{}_X Z - \nabla^*{}_{[X,Y]}Z, \tag{3.1}$$

where ∇^* is the Schouten-van Kampen connection. In view of equation (2.16), equation (3.1) takes the form

$$R^*(X,Y)Z = R(X,Y)Z + \epsilon[g(Y,Z)X - g(X,Z)Y], \tag{3.2}$$

where

$$R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z, \tag{3.3}$$

is the Riemannian curvature tensor of Levi-Civita connection ∇.

Here equation (3.2) is the relation between Riemannian curvature tensor with respect to Schouten-van Kampen connection ∇^* and Levi-Civita connection ∇.

Transvection of V in equation (3.2), gives

$$R^*(X,Y,Z,V) = R(X,Y,Z,V) + \epsilon[g(Y,Z)g(X,V) - g(X,Z)g(Y,V)], \tag{3.4}$$

where

$$R^*(X,Y,Z,V) = g(R^*(X,Y)Z,V)$$

and

$$R(X,Y,Z,V) = g(R(X,Y)Z,V).$$

Putting $X = V = e_i$ in equation (3.4) and taking summation over i, $1 \leq i \leq n$, we get

$$S^*(Y,Z) = S(Y,Z) + \epsilon(n-1)g(Y,Z), \tag{3.5}$$

where S^* and S denotes the Ricci tensors with respect to the connections ∇^* and ∇ respectively. Again putting $Y = Z = e_i$ in equation (3.6) and taking summation over i, $1 \leq i \leq n$, we get

$$r^* = r + \epsilon n(n-1), \tag{3.6}$$

where r^* and r denotes the scalar curvatures with respect to connections ∇^* and ∇ respectively.

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